# GPS Theory and Positioning

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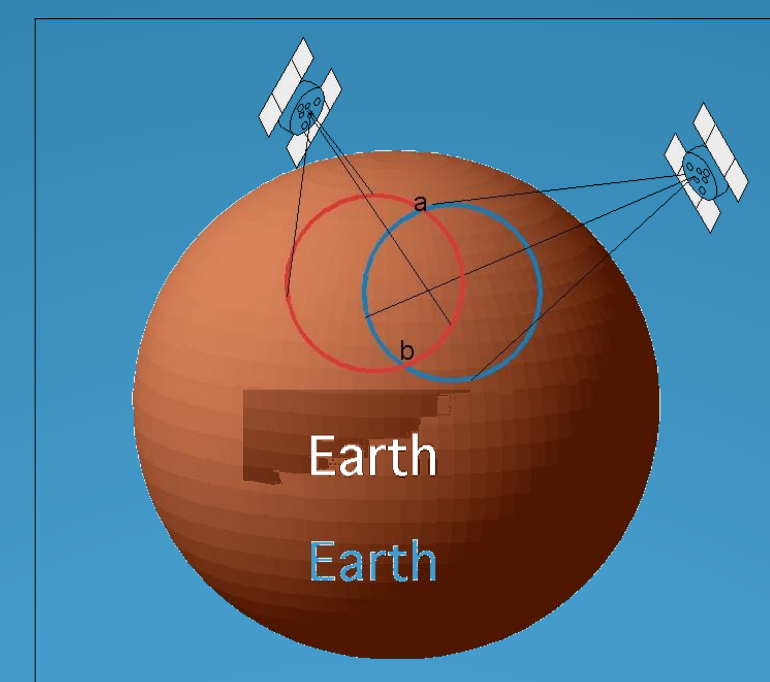
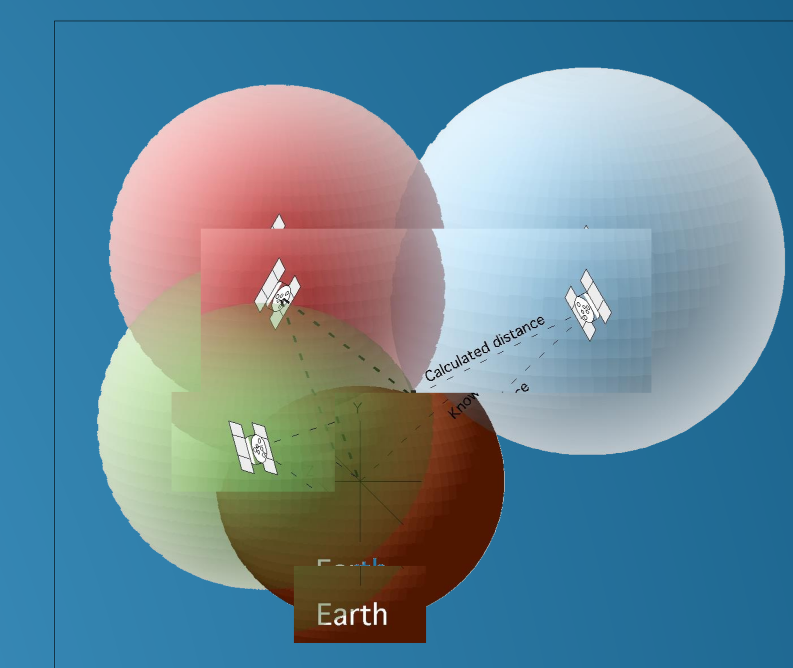
INTRODUCTION
GPS has many uses, and of them positioning is one of the most widely used. GPS's make use of incredibly well synchronized clocks. There are 24 satellites that are constantly orbiting the earth. We use the signals from these satellites to accurately determine position



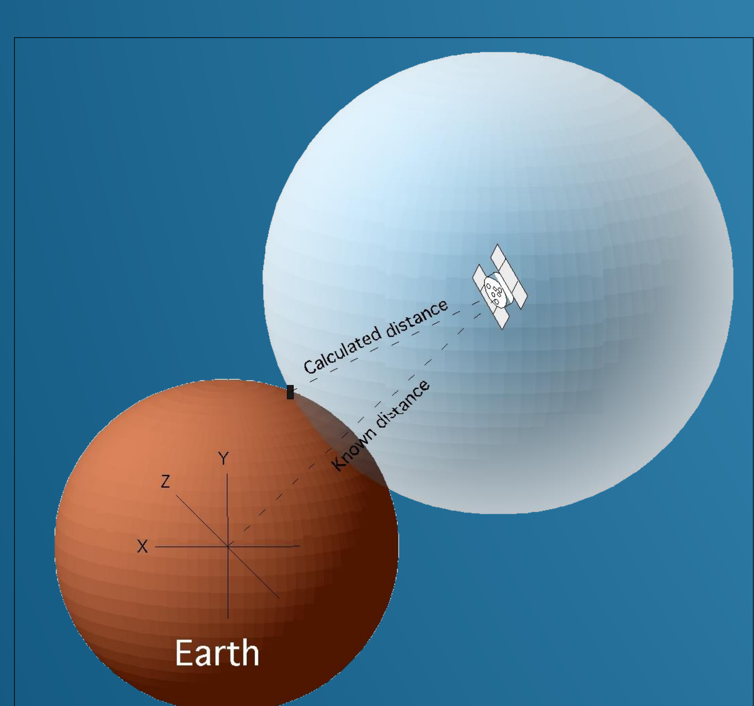

With signals from two satellites, the receiver can narrow down its location to just two points on the earths surface. Were the two circles intersect.

Knowing its distance from three satellites,a receiver can pin its location to just two points, and one of the points is out in space




The general pseudorange equation is given as

$$\rho_r^s = (t_r , t_s)c - (\delta t_r - \delta t_s)c$$




One satellite gives the position within a circle of an area proportional to the distance of the satellite to the receiver

To improve the accuracy of the GPS positioning algorithms the time must be measured as precisely as possible. Here relativistic effects are taken into account

$$\delta t' = \delta t/(1 - v^2/c^2)^{1/2}$$

$$(\delta t' - \delta t)/\ \delta t' = \Delta U/c^2$$

Each satellite knows its position and its distance from the center of the earth.
Each satellite constantly broadcasts this information.
With this information and the calculated distance, the receiver calculates its position.
Just knowing the distance to one satellite doesn't provide enough information.

Major sources of noise are from the ionosphere, the troposphere and noisy GPS receivers

The pseudorange model is now

$$\rho_r^s = (t_r , t_s)c - (\delta t_r - \delta t_s)c + \delta_{ion} + \delta_{tro} + \delta_{mu} l\ \delta + \delta_{rel} + \epsilon$$

A fourth satellite is used to determine the final point

To solve for position we use a nonlinear least squares algorithm

$\rho 1(x, y, z, d) = [ (x-A_1)^2 + (y-B_1)^2 - (z-C_1)^2 ]^{1/2} - c(t_1-d) = 0$
$\rho 2(x, y, z, d) = [ (x-A_2)^2 + (y-B_2)^2 - (z-C_2)^2 ]^{1/2} - c(t_2-d) = 0$
$\rho 3(x, y, z, d) = [ (x-A_3)^2 + (y-B_3)^2 - (z-C_3)^2 ]^{1/2} - c(t_3-d) = 0$
$\rho 4(x, y, z, d) = [ (x-A_4)^2 + (y-B_4)^2 - (z-C_4)^2 ]^{1/2} - c(t_4-d) = 0$

## References

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