Modeling the Cost Effectiveness of Fire Protection Investment and Resource Allocation in the US: Models and a 1980-2011 Case Study

Adam Behrendt, Vineet Madasseri Payyappalli, Dr. Jun Zhuang
Department of Industrial & Systems Engineering, University at Buffalo – The State University of New York

Motivation and Introduction

Fire-related hazards and incidents are an everyday phenomenon, and firefighting in the United States owe to more than one million firefighters in about 30,000 fire departments across the country [1]. The estimated total cost of fire was $329 billion in 2011 [2], and yet there is little work in the literature about risk assessment, cost-benefit analysis, and resource allocation in fire protection. Leveraging the data from the National Fire Protection Association (NFPA) reports, we conduct a data-driven study to propose empirical and theoretical models to assess risk levels, develop risk-reduction strategies and estimate the effectiveness of investment in fire protection. An optimal resource allocation model with equity considerations and its numerical analyses provide insights and techniques to ease the decision-making process which features the trade-off between equity and efficiency.

Data Analysis[3]

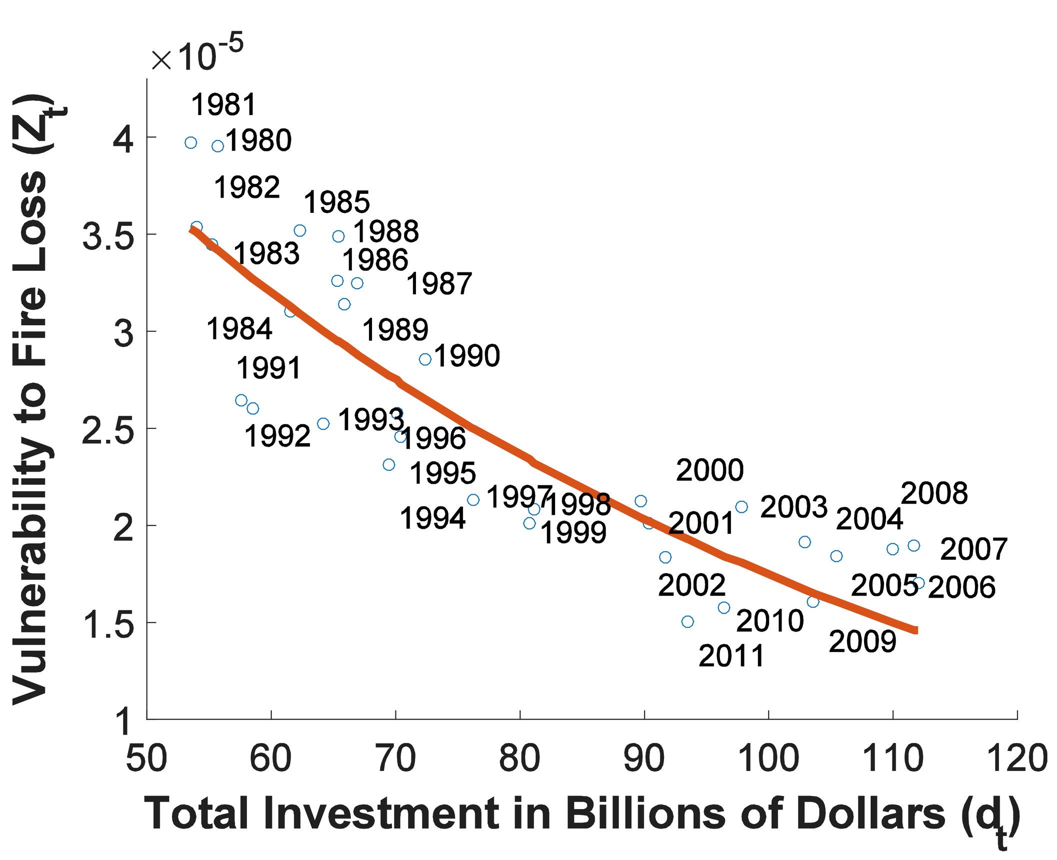
In the first stage of this project a relationship between loss and investment in fire protection was found by analyzing existing data and using exponential regression to describe this relationship.

Vulnerability $Z_t = \frac{q_t}{Q_t} = a_o e^{-\lambda_0 d_t}$.

Notation:

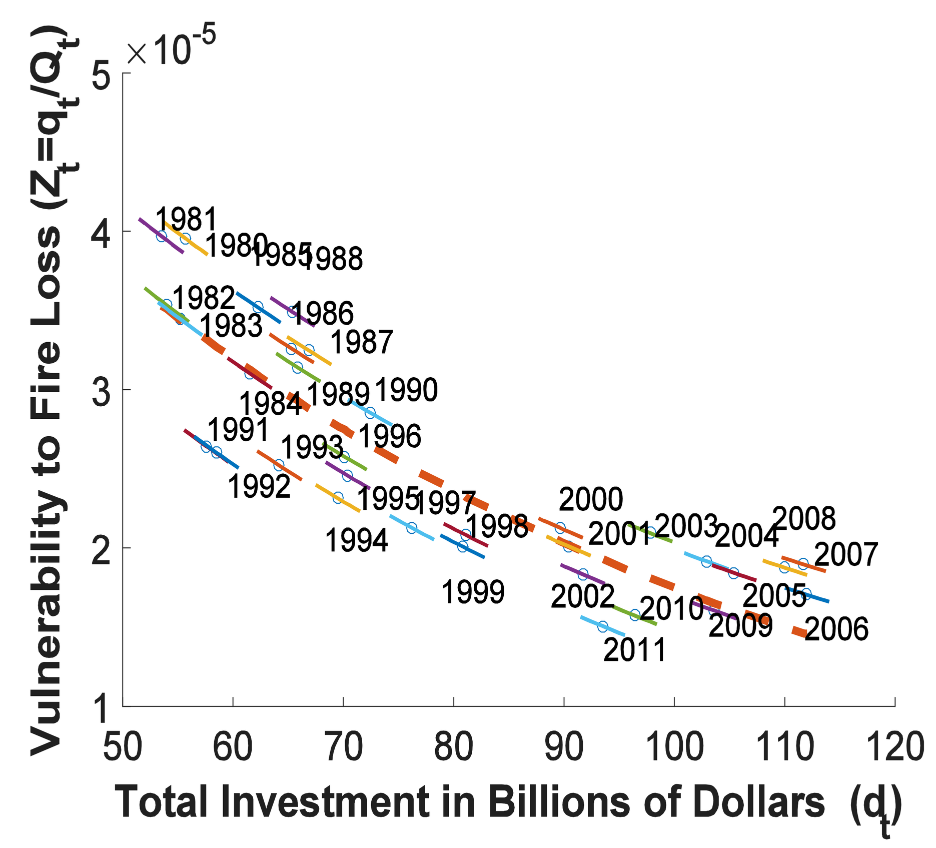
q_t: actual loss, Q_t: potential loss, t: year,
a_0: initial vulnerability, λ_0: investment effectiveness,
d_t: fire protection investment.

Results show high R^2 value (0.8) between Z_t and d_t for 1980-2011 period.



Investment effectiveness[3]

Due to the nature of a best fit line, there remains left over variability at data points that do not directly fall on the line. Therefore we create individual best fit lines by equating the first derivatives of the individual lines with the overall regression line and re-defining our vulnerability equation for vector based parameters.



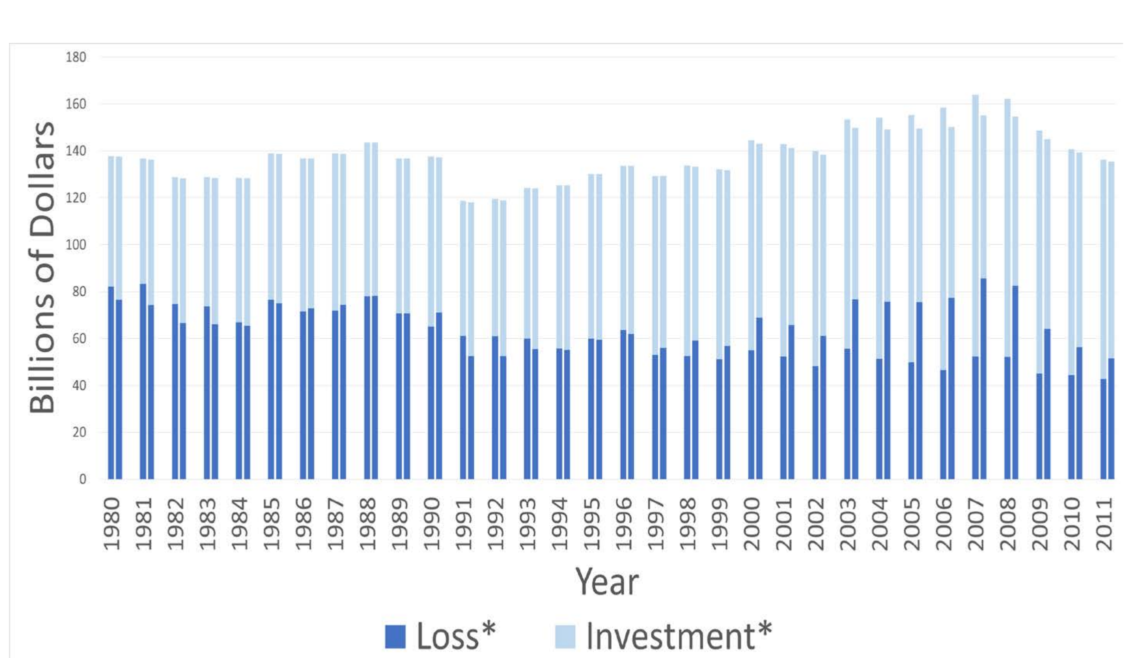
We can then estimate regression parameters, namely initial vulnerability a_t and investment effectiveness λ_t for each year using:

$$\lambda_t = \frac{a_o \lambda_o e^{-\lambda_0 d_t}}{Z_t} \quad \text{and} \quad a_t = Z_t e^{\left(\frac{a_o \lambda_o e^{-\lambda_0 d_t}}{Z_t}\right)}$$

Using the above, we generate an expression for total cost (U_t) which is minimized as follows:

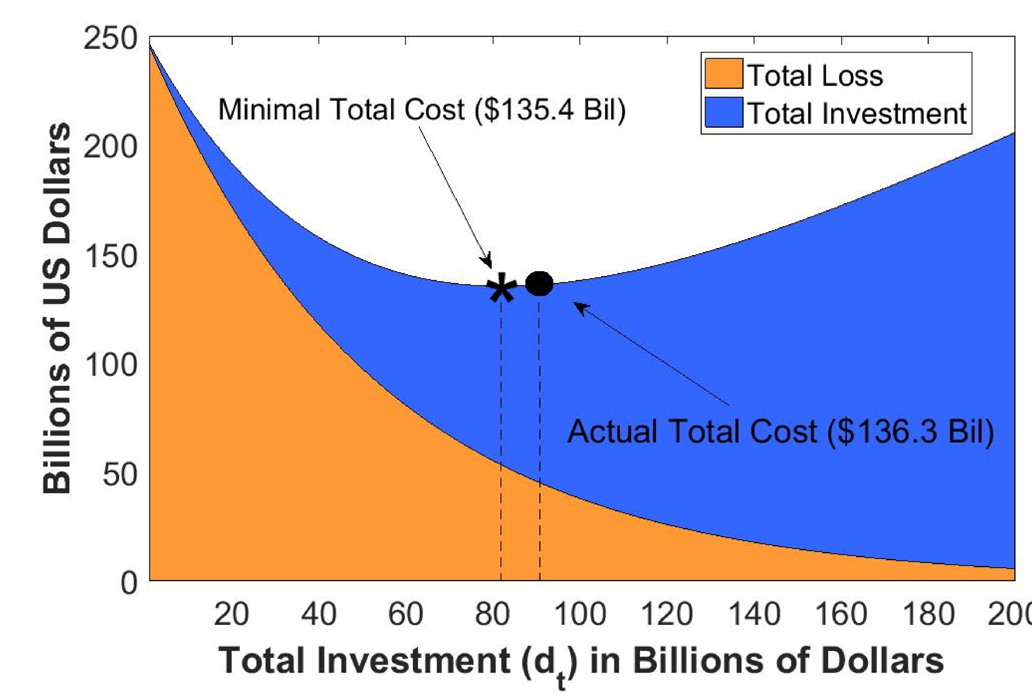
$$\min_{d_t} U_t = Q_t a_t e^{-\lambda_t d_t} + d_t; \quad d_t \geq 0$$

As shown below we can now use this expression using actual data from each year from 1980-2011 to solve for the optimal level of investment each year that would have led to the minimal total cost of fire for that year.



Investment effectiveness[3] Cont.

Presented here is a closer look at the actual and optimal cost breakdown of 2011:



Optimal resource allocation[3]

An optimal resource allocation (RA) formulation (for example, see [4]) is useful for a decision maker who allocates limited resources across multiple regions/fire stations.

Decision maker: Government/organization

Objective: $\min_{d_i} \sum_{i=1}^{n} g_i a_i e^{-\lambda_i d_i}$

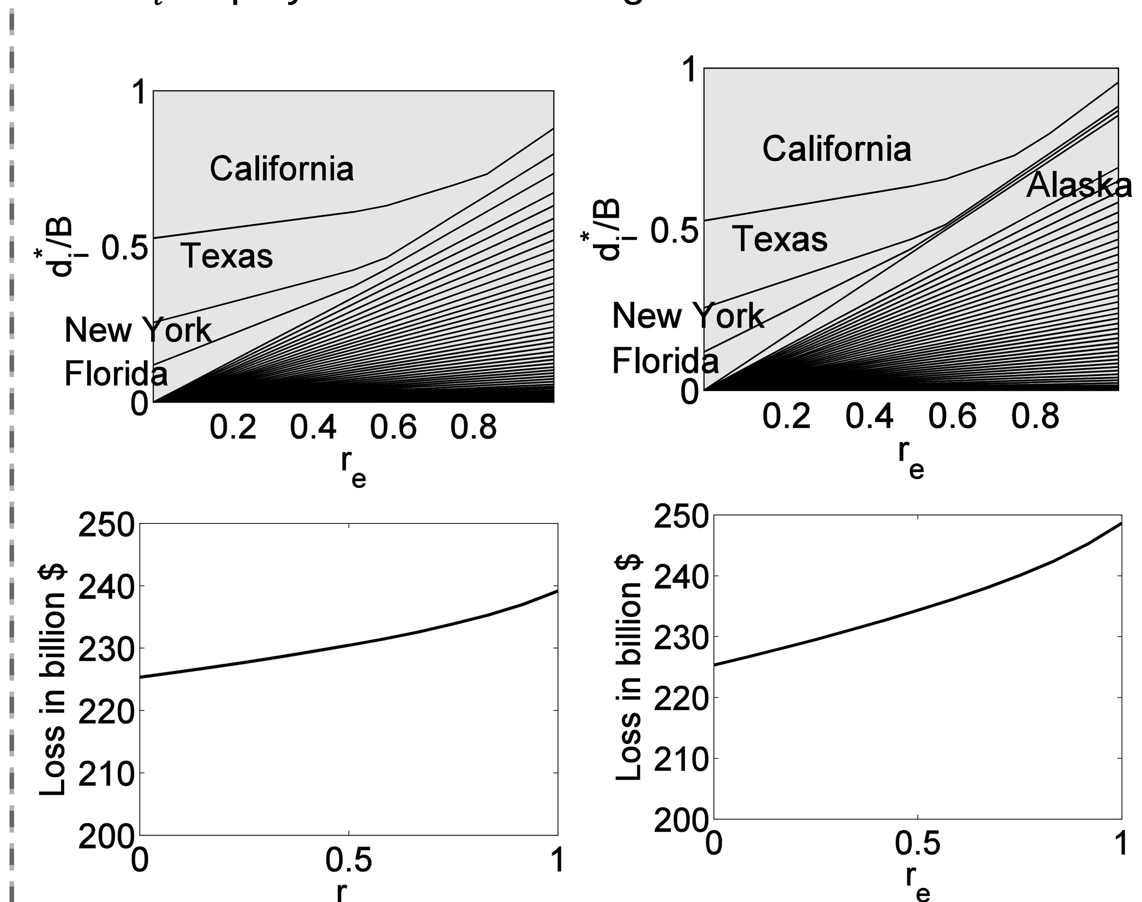
Constraints: $\sum_{i=1}^{n} d_i \leq B$ and $d_i \geq d_i^e \; \forall i$
where $d_i^e = c_i r_e B$ and $\sum_{i=1}^{n} c_i = 1$

Decision variable:

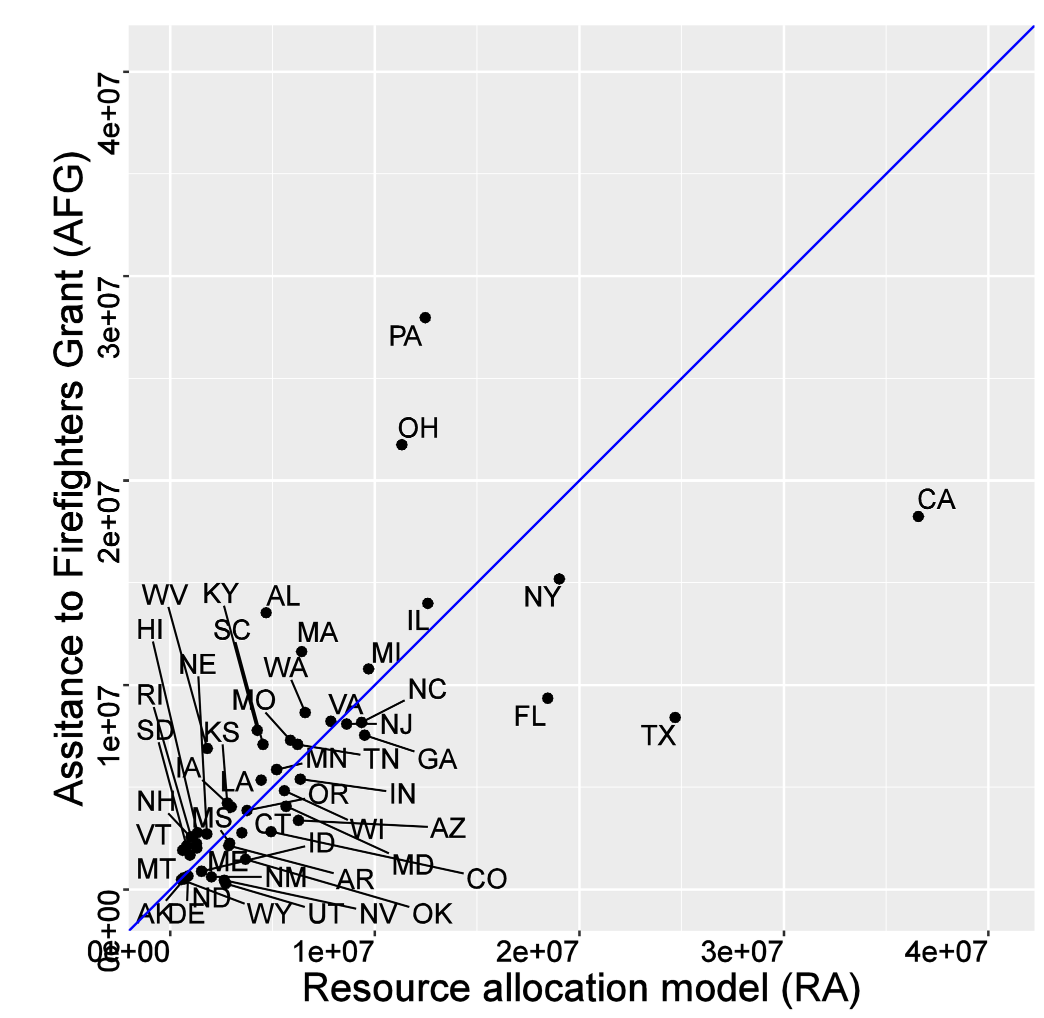
d_i: fire protection investment in region $i \in R$

Parameters:

$R = \{i: i = 1,2,\dots,n\}$: set of regions
n : number of regions
g_i: potential loss of region i
a_i: initial vulnerability of region i
λ_i: investment effectiveness of region i
B : available budget
d_i^e: equity-based allocation to region i
r_e: equity level
c_i: equity coefficient of region i



Comparison of RA model with actual federal fire grant allocation [5] to infer potential overspending and underspending in certain states.



Conclusions

- Investment in fire protection has an exponential-decay relationship with the nation's vulnerability to fire
- Minimal levels of the total cost may result in higher levels of loss due to the decreasing marginal effectiveness of investment
- Initial vulnerability decreases and investment effectiveness increase over the years, showing the compounding effects of investment and fire protection advancement, respectively
- Tradeoffs exist between efficiency and equity in how funds are distributed to more efficient states or more proportionally among all states
- Some states may over/under invest in fire protection

References

[1] H. J. G. Haynes and G. P. Stein (2016), "U.S. Fire Department Profile – 2014", NFPA Report.
[2] J. R. Hall. Jr. (2014), "The Total Cost of Fire in the United States", NFPA Report.
[3] A. Behrendt, V. M. Payyappalli, J. Zhuang (working paper). Modeling the Cost Effectiveness of Fire Protection Resource Allocation in the United States: Models and a 1980-2011 Case Study.
[4] X. Shan, and J. Zhuang (2013), "Cost of Equity in Homeland Security Resource Allocation In the Face of A Strategic Attacker", Risk Analysis, 33(6): 1083-1099.
[5] L. G. Kruger (2016). Assistance to Firefighters Program: Distribution of fire grant funding. Congressional Research Service (CRS) Report.